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On the presentation of ship's hull girder section modulus in probabilistic formats

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Abstract

Traditionally, the probabilistic analysis for wave-induced hull girder bending stresses is normally evaluated based on the so-called 'annual' probability density function (PDF) of hull girder section modulus. However, the design wave-induced loads given by the Class Rules are specified for the whole design life of a ship. To solve this problem, a kind of PDF of hull girder section modulus taking into account 'time period' was introduced in the paper. A parametric study was performed to investigate the effects of the two types of PDFs of hull girder section modulus, namely the 'annual' and 'time period' PDFs, on the PDF of the wave-induced hull girder bending stresses. The results show that the probability of exceeding given permissible hull girder bending stress calculated

using the ‘time period’ PDF of hull girder section modulus is smaller than that calculated based on the ‘annual’ PDF of hull girder section modulus. Also, a palpable difference between the PDFs of wave-induced hull girder bending stresses (based on the ‘annual’ and ‘time period’ PDFs of hull girder section modulus) may occur after ship’s age of 15 years.

Keywords: ship’s hull girder section modulus; vertical wave-induced bending moments (VWBM); Hull girder bending stresses; probability density functions (PDFs)

Nomenclature

BM	=	Bending Moment
C	=	Coefficient to multiply the ordinates of given truncated PDF
CDF	=	Cumulative Distribution Function
COV	=	Coefficient of Variance
F	=	Cumulative Distribution Function
f	=	Probability Density Function
P	=	Probability
PDF	=	Probability Density Function
POE	=	Probability of Exceedance of any given value
r	=	Correlation coefficient
s	=	Bending stress
T	=	Time (ship’s service life or any year of it)
T_s	=	Ship’s service life
V	=	Space below the envelope of ‘annual’ PDFs
VWBM	=	Vertical Wave-Induced Bending Moments
Z	=	Ship’s Hull Girder Section Modulus

Z_d	=	Hull Girder Section Modulus at ship's deck
$Z_{d,T}$	=	Hull Girder Section Modulus at ship's deck for year T
$Z_{min, IMO}$	=	Minimal Ship's Hull Girder Section Modulus required by IMO
Z_{nom}	=	Nominal Ship's Hull Girder Section Modulus (i.e., calculated with data from drawings)
ΔT	=	Interval between two times
α	=	Scale parameter of Weibull distribution
λ	=	Shape parameter of Weibull distribution
σ	=	Standard deviation

1. Introduction

The hull girder section modulus of a ship changes over time mostly due to corrosion. Since the loss of the plates' and framing's thickness due to corrosion is a random value, the hull girder section modulus is also a random parameter with a certain probability distribution. The research on uncertainty analysis for geometric properties of ship's hull structures is relatively new subject compared to the uncertainty evaluation of the hydrodynamic loading acting on ship's hull structures. Although the uncertainties in geometric properties of hull structures are not of primary importance, there are cases when more accurate presentation of the hull girder geometric properties in probabilistic format is necessary. This is especially true while assessing the structural strength of aging ships. Therefore, the consideration of the uncertainties of the geometric properties of hull structures leads to more rational evaluation for the hull girder strength.

Considerable work has been conducted on uncertainty evaluation for hull girder section modulus. Guedes Soares and Garbatov (1996, 1997, 1998, 2002) and Garbatov and Guedes Soares (2008, 2011) assumed the hull girder section modulus as a random variable following a normal distribution in hull girder reliability analysis. Ivanov (2007, 2012a, 2012b) and Ivanov et al. (2007) investigated the type of probabilistic distribution of hull girder section modulus over ship's service time according to the data of corrosion wastage of ship structures published by Paik and Park (1998). Uncertainty analysis for geometric properties of stiffeners due to corrosion

was studied by Ivanov (2008), Silva et al (2014), and Chen (2017) for more precise prediction of the local strength of ship structures. Ivanov and Kokarakis (2007) performed deterministic and probabilistic calculation of the geometric properties of corrugated bulkheads of different ship types accounting for the corrosion effects. Several corrosion models have also been developed by Guedes Soares & Garbatov (1999), Paik et al. (2004), Yamamoto & Kobayashi (2005), and Paik and Kim (2012) to describe the corrosion phenomena of marine structures.

Recently, Zayed et al. (2013) conducted a reliability assessment for ship hulls subjected to corrosion and maintenance and hull girder strength was assessed based on the corrosion model proposed by Guedes Soares & Garbatov (1999). Chen (2017) performed panel reliability assessment for FPSOs and corrosion waste of panels was evaluated based on a Rule-based corrosion model. Campanile et al. (2015) investigated the residual hull girder strength of bulk carriers and the effect of corrosion wastage on hull girder strength was evaluated by Monte Carlo simulation. Chen (2016) proposed a methodology for FPSO hull girder reliability assessment and the rule-based corrosion model was used to evaluate the time-dependent hull girder ultimate strength.

The minimum ship's hull girder section modulus required by Classification Societies (ABS, 2016) is usually determined by the design vertical wave-induced bending moment (VWBM). According to the "permissible stresses" approach, the nominal stresses are calculated and compared with the permissible stresses given by the Class Rules. For example, the effect of VWBM on ship's hull girder strength is calculated as the ratio between the design VWBM and hull girder section modulus.

Traditionally, the probabilistic analysis for wave-induced hull girder bending stresses is normally evaluated based on the so-called 'annual' probability density function (PDF) of hull girder section modulus. However, the design wave-induced loads given by the Class Rules are specified for the whole design life of a ship. To solve this problem, a kind of PDF of hull girder section modulus taking into account 'time period' is introduced in the paper. A parametric study is performed to investigate the effects of the two types of PDFs of hull girder section modulus, namely the 'annual' and 'time period' PDFs, on the PDFs of the wave-induced hull girder bending stresses.

2. Probabilistic distribution of hull girder section modulus

The hull girder section modulus Z changes over ship's service life mostly due to corrosion. Since the loss due to corrosion is a random value, Z is also a random parameter with its own probabilistic distribution. Bearing in mind that the design VWBM refers to the given time period (e.g., ship's service life), the probabilistic distribution of Z should be valid for the same time period as for the design VWBM. This probabilistic distribution of Z can be derived by following these steps:

- First, build the 'annual' probability density functions (PDFs) of Z considering the reduction of Z due to corrosion wear for several years (e.g., at year five, year six, year seven, etc.). In the calculations of the PDFs of Z , the probabilistic distributions of the following input parameters are used: PDFs of the thicknesses of as-rolled plates and the geometric properties of shipbuilding structural profiles are proven to follow normal distribution Ivanov (2007, 2008); The annual corrosion rate of plates and structural profiles for bulk carriers follows Weibull distribution as proven by Paik and Park (1998); The "annual" PDFs of the hull girder section modulus follow normal distribution as shown in Campanile et al (2015), Ivanov (2007), Ivanov (2012a) which was also confirmed by Monte Carlo simulation (Ivanov et al., 2007).

Truncation of PDF is introduced. The physical reasons for truncation are based on the fact that as-rolled plates' thicknesses cannot be infinite, the corrosion wastage cannot be greater than the plates' thicknesses; and the hull girder section modulus cannot be infinite or smaller than 90% of the as-built hull girder section modulus according to the requirements of the International Maritime Organization (IMO, 2000). The type of PDFs of Z slightly changes due to truncation but the parameters of PDFs (e.g., mean values, standard deviations, etc.) change (Ivanov, 2012b; Ivanov and Kokarakis, 2007), which was also taken into consideration in the work done.

In the example here a 25000 tones deadweight bulk carrier was used as a sample ship. It was assumed that: there is no corrosion during the first three years of ship's operational life due to the effect of coating protection; gradual worsening of the coating protection happens during the next two

years; and the annual corrosion rate until the end of the ship's service life (in the example – 25 years) is a constant.

The quotation mark is used to emphasize the fact that the Probability Density Functions (PDFs) are only valid for the last year of any given time period (see, e.g., Fig. 1).

- Put an envelope over the 'annual' PDFs (see Fig. 2). In this case, the space 'V' below the envelope of the PDFs is equal to the given time period (e.g. service life of 25 years) because the space 'V' is equal to the product of 'one' (i.e. the area below the PDF as a basis of the figure) multiplied by the time period. To keep the format of the calculations close to the traditional format of the PDFs, one can assume that the space below the envelope is equal to unity. Thus, the space is equal to the area of the rectangle with length T and height h = 1 shown in Fig. 3. The length of Fig. 3 is the ships' age span, T, and the height is equal to unity because the cross-sectional area below each PDF is equal to unity, i.e.,

$$V = 1 \times T = 1(T_n - T_0) = 1 \times \Delta T \times n \quad (1)$$

where n = the year for the last 'annual' PDF, ΔT = length of the equal intervals into which the ships' age span (i.e. $T_n - T_0$) is broken up (see Fig. 3 and Fig. 4).

- Specify the value of Z beyond which the vessel is viewed not fit for the intended service. This specified value is the minimal permissible value of Z (in merchant ships – 90% of the as-built hull girder section modulus according to IMO requirements (IMO, 2000)).
- Calculate the space below the envelope of the 'annual' PDFs between zero and the permissible value of Z. This space, V, should be equal to the area of Fig. 4. It has been calculated by the trapezoidal rule:

$$V_i = \Delta T \left[\sum_{i=0}^n h_i - \frac{1}{2}(h_0 + h_n) \right] \quad (2)$$

where h_i = cross-sectional area below any ‘annual’ PDF aft of the assumed value of Z ; h_0 = the first cross-sectional area at “zero” cross-section; h_n = the last cross-sectional area at section through the specified value of Z .

By definition, these cross-sectional areas are equal to the ordinate h_i of the cumulative distribution function (CDF) for the corresponding ‘ith’ year.

- Build the CDF for the whole ship’s age span. The value of the ordinate of the CDF that corresponds to any assumed value of Z is calculated by division of the space determined by Eq. (2) by the total space determined by Eq. (1), i.e.,

$$\frac{V_i}{V} = \frac{\sum_{i=0}^n h_i - \frac{1}{2}(h_0 + h_n)}{n} \quad (3)$$

Note: The CDF can be calculated not only for the whole ship’s service life but also for any given time period.

- Create the PDF for the whole ship’s age span (or any other given time period) by numerically differentiating the so-derived CDF. In geometric terms, the value of the PDF at any given Z is equal to the tangent of the angle between the tangent line at any given Z and the abscissa axis. The accuracy of the calculated PDF is also checked by integrating it to derive the area below the curve. This area should be equal to unity, and this is the criteria used to assess the accuracy of the procedure. The error resulting from application of the trapezoidal rule and the numeric differentiation of the CDF is smaller than 1%. The so-derived PDFs of Z for the sample bulk carrier are shown as an example on Fig. 5.

Further, the so-derived PDFs are named as ‘time period’ PDFs of Z . It should also be emphasized here that they are not an algebraic average of the ‘annual’ probabilistic distributions. The reason is that the corrosion does not start immediately when the ship is built. There is a period of time when the anti-corrosion protection is still working. Within that period the ‘annual’ PDFs do not change. The randomness of Z at this period is due only to

the random nature of the geometric properties of as-rolled plates and as-rolled (or built-up) structural profiles. As an example, Fig. 2 (on the axis for ship's age) shows the length of this period from zero to three years (see also Ivanov, 2007; Ivanov and Kokarakis, 2007).

3. Comparison of the two types of PDFs of Z

From the above said, it follows that the 'annual' PDFs of Z and 'time period' PDFs of Z are two different types of PDFs. To illustrate the differences between them they are shown in Fig. 6 – Fig. 10 for several pairs of PDFs. One can observe the fact that the difference for time period $T = 0 - 5$ years is negligible but after that increases quite substantially with the increase of the ship's age. Therefore, the application of 'time period' PDFs of Z in strength calculations of old ships provides greater accuracy in the calculations than when 'annual' PDFs of Z are applied. The procedure applied for calculation of the PDFs of wave induced hull girder bending stresses using the two types of PDFs is given below.

4. PDFs of the VWBM for different time periods

The calculation of the PDFs of bending stresses due to VWBM was performed for the same time (or time period). That required presentation of the VWBM in probabilistic format for the same time period as for the 'time period' PDFs of Z. This was done following the methodology described in Ivanov (2009) and Kamenov-Toshkov et al. (2006).

The major assumption in derivation of the shape and scale parameters of Weibull distribution of WBM is that the shape and scale parameters change over different time periods but the PDF is always Weibull distribution as shown in Eq. (4). As explained in Ivanov (2007) and Ivanov (2009), the reason for this change is the fact that the ship is exposed to different number of wave actions within different time periods.

The results of the calculations for the sample 25K DWT bulk carrier used in the example here are shown in Table 1. The PDFs of VWBM for several time periods are shown in Fig. 11. The design VWBM for this bulk carrier was calculated with the formulas in ABS Rules (ABS, 2016) and found to be 94795 t.m.

$$f(x) = \frac{\lambda x^{\lambda-1}}{\alpha^\lambda} e^{-\left(\frac{x}{\alpha}\right)^\lambda} \quad (4)$$

The bending stresses 's' in the deck of the sample bulk carrier are calculated by the formula

$$s = \frac{\text{design VWBM}}{Z_{\text{deck}}} \quad (5)$$

Table 1 Shape and Scale Parameters of Weibull distribution of VWBM for the sample 25,000 tones deadweight bulk carrier (the parameter α is in t.m)

Year	λ	α	Year	λ	α
1	0.6559	909	14	0.7736	2192
2	0.6832	1150	15	0.7773	2242
3	0.7002	1317	16	0.7807	2290
4	0.7128	1450	17	0.7840	2336
5	0.7230	1562	18	0.7871	2380
6	0.7314	1659	19	0.7901	2422
7	0.7388	1746	20	0.7929	2463
8	0.7452	1824	21	0.7956	2502
9	0.7510	1897	22	0.7982	2540
10	0.7563	1963	23	0.8007	2577
11	0.7611	2026	24	0.8032	2613
12	0.7656	2084	25	0.8055	2647
13	0.7697	2140			

Since all calculations were planned to be performed in probabilistic format, the convolution integration method was used for equation (5) to obtain the PDF of bending stresses for several time periods (i.e., 0 - 5, 0 - 10, 0 - 15, 0 - 20, and 0 - 25 years), i.e.

$$f(s) = \int_0^\infty Z f(sZ) f(Z) dZ \quad (6)$$

where $f(s)$ = PDF of s , $f(sZ)$ = PDF of design VWBM where any value of the design VWBM is calculated as the product of s and Z , $f(Z)$ = PDF of Z .

Equation (6) is valid for independent parameters (in this case the design VWBM and hull girder section modulus Z). For any given time period (as given above), two types of the numerical calculations were performed – once with ‘annual’ and then – with ‘time period’ PDF of Z .

The PDF of bending stresses is very close to exponential distribution as can be seen in Fig. 12 (since the two curves are too close, the same figure is presented in Fig. 13 using log-scale of the axes). Once the PDFs of bending stress, s , are built, the probability of exceedance of any given value can be calculated.

Table 2 contains the numerical results for the probabilities of exceedance, POE, of the assumed permissible bending stress when the two types of PDFs of the hull girder section modulus, Z , are used. The PDF of VWBM is Weibull two-parameter distribution (the parameters for several time periods are given in Table 1).

To obtain the PDF of the bending stress due to VWBM, the PDF of Weibull distribution is divided by the “time period” PDF of Z and the “annual” PDF of Z , respectively. Thus, two PDFs of bending stresses are derived: one for the case when “time period” PDF of Z is used and another one for when the “annual” PDF of Z is used. Once the PDF of bending stresses is determined, the POE of any given permissible value (in the example, the assumed permissible bending stress is 1340 kg/cm²) can be determined.

The last row of Table 2 contains the ratio between the POE of the assumed permissible bending stress derived when the two types of PDFs are used. The numerical results for the example are also illustrated in Fig. 14 where the noticeable difference can be observed after ship’s age of 15 years.

Note: There is no formula in Classification Societies Rules for the permissible bending stresses due to VWBM. Therefore they were assumed as the difference between bending stresses due to total BM and bending stresses due to still water BM in port (i.e., 1340 kg/cm²). However, permissible bending stress due to design WBM can be any other value depending on the case. The assumed permissible bending stress in the example here is used only to illustrate the effect of the two types of PDFs of Z on the PDFs of VWBM induced bending stresses. The same approach can also be applied to show the effect of the two types of PDFs of Z on the PDFs of bending stresses induced by Still Water Bending Moment or the total hull girder Bending Moment.

Table 2 Results of the calculations for the effect of different PDFs of Z on WBM induced bending stresses

year T or time period ΔT	5	10	15	20	25
P (s > [s]) with ‘annual’ PDF of Z	7.10E-11	1.60E-10	3.05E-10	5.51E-10	8.81E-10
P (s > [s]) with ‘time period’ PDF of Z	6.69E-11	1.28E-10	2.06E-10	3.15E-10	4.59E-10
<u>POE of s derived with ‘time period’ PDF of Z</u> POE of s derived with ‘annual’ PDF of Z	0.942	0.800	0.675	0.572	0.521

Note: [s] = permissible bending stress (in the example – 1340 kg/cm²), P (s > [s]) = Probability of Exceeding given permissible bending stress.

The numeric values of P (s > [s]) and the corresponding ratios of POE are given in scientific numbers. They clearly show that the calculations of P (s > [s]) with the ‘time period’ PDF of Z lead to smaller values of P (s > [s]) than in the calculations of P (s > [s]) with the ‘annual’ PDF of Z. It results from the fact that the majority of area below the ‘time period’ PDF of Z is shifted towards greater values of Z (see Fig. 7 – Fig. 10) than in the ‘annual’ PDF of Z which is a symmetric Gaussian distribution.

5. Notes on the term ‘annual’ PDFs of Z

In general, the term ‘annual’ is used in many cases where the deterministic approach is applied. For example, it is used to calculate the corrosion wastage of steel plates by multiplying the annual corrosion rate by the corresponding time period (in years) under consideration. The essence of this straight forward approach is the assumption that the corrosion wastage of steel plates is a linear function of time, which means that the corrosion rate at any year is a constant value. Similar situation exists for the section modulus of shipbuilding structural profiles and for the hull girder section modulus reduction over ship’s service life when deterministic method is applied. If the value of Z_i at ‘i-th’ year is known, the value of Z_{i+1} for the following ‘i-th+1’ year could be calculated by the simple formula

$$Z_{i+1} = Z_i - \Delta Z \quad (7)$$

where ΔZ = quantity lost by diminution of Z (due to corrosion) assumed as a constant value after the time when coating longevity has been reached.

When a probabilistic method is used in the calculations of the hull girder section modulus, its mean and standard deviation have to be calculated for any year or time period (Table 3 contains these data for the sample bulk carrier). The annual decrement of Z for its mean values and the annual increment of Z for its standard deviations over ship's service life are given in Table 4. Based on the data in Table 4, one can conclude that the annual decrement of the mean value of Z (after coasting breaks) practically does not change which confirms the dependency between ship's age and the reduction of the mean value of Z (due to corrosion) as a linear one. One can also notice that the relationship between the standard deviations of Z and ship's age does not follow linear dependency.

Table 3 Mean values and standard deviations of “annual” PDF of Z of the sample 25K DWT bulk carrier for each year of ship's service life

Year	mean	st. dev.	COV	Coef. C	Year	mean	st. dev.	COV	Coef. C
	m.cm ²	m.cm ²	[-]	[-]		m.cm ²	m.cm ²	[-]	[-]
0	93435	997	0.0107	1.0097	13	91183	2219	0.0243	1.0197
1	93435	997	0.0107	1.0097	14	90937	2403	0.0264	1.0226
2	93435	997	0.0107	1.0097	15	90692	2590	0.0286	1.0256
3	93435	997	0.0107	1.0097	16	90446	2779	0.0307	1.0285
4	93292	1032	0.0111	1.0082	17	90200	2971	0.0329	1.0315
5	93149	1075	0.0115	1.0074	18	89954	3164	0.0352	1.0346
6	92903	1155	0.0124	1.0066	19	89709	3358	0.0374	1.0380
7	92658	1263	0.0136	1.0069	20	89463	3554	0.0397	1.0420
8	92412	1393	0.0151	1.0080	21	89217	3751	0.0420	1.0469
9	92166	1540	0.0167	1.0097	22	88972	3948	0.0444	1.0529
10	91920	1698	0.0185	1.0118	23	88726	4146	0.0467	1.0605
11	91675	1865	0.0203	1.0143	24	88480	4345	0.0491	1.0697
12	91429	2040	0.0223	1.0169	25	88235	4544	0.0515	1.0808

Notes:

- COV = coefficient of variance (st. deviation / mean),

- The coefficient C is related to the changes of the PDFs change due to truncation. It is used to multiply the ordinates of given truncated PDF in order to fulfill the requirement that the area below PDF should be equal to unity. The starting point for finding the equation of the ordinate of the new truncated normal distribution function of Z_d is the premise that the area below the truncated normal distribution within the two boundaries b_u and b_l should be equal to unity (as for a normal distribution with boundaries $-\infty, +\infty$). The difference between the ordinates of the two probability density functions will be a constant C (see Eq. 8) that could be determined the following way (Ivanov, 2007):

$$\int_{b_l}^{b_u} f_c(Z_d) dZ_d = C \int_{b_l}^{b_u} f(Z_d) dZ_d = C [F(b_u) - F(b_l)] = 1 \quad C = 1 / [F(b_u) - F(b_l)] \quad (8)$$

where b_u = given upper truncation (i.e. the maximal value of Z_d), b_l = given lower truncation (i.e., the minimal value of Z_d), F = CDF.

Thus, the truncated normal distribution function of Z_d , $f_c(Z_d)$, will be:

$$f_c(Z_d) = \frac{1}{\Phi\left(\frac{b_u - \bar{Z}_d}{\sigma_{Z,d}}\right) - \Phi\left(\frac{b_l - \bar{Z}_d}{\sigma_{Z,d}}\right)} f(Z_d); \quad f(Z_d) = \frac{1}{\sigma_{Z,d} \sqrt{2\pi}} \exp\left[-\left(\frac{Z_d - \bar{Z}_d}{\sigma_{Z,d} \sqrt{2}}\right)^2\right] \quad (9)$$

where \bar{Z}_d = mean value of Z_d , $\sigma_{Z,d}$ = standard deviation of Z_d , Φ = Laplace integral.

For given time 'T', the probability of Z_d being between maximal and minimal boundaries $Z_{d,u}$ and $Z_{d,l}$ will be

$$P(Z_{d,l} \leq Z_d \leq Z_{d,u}; T) = C_T \int_{Z_{d,l}}^{Z_{d,u}} f_T(Z_d) dZ_d \quad (10)$$

where C_T is the coefficient determined with eq.(9) for given time T.

Table 4 Decrement for the mean values and increment for the standard deviations of Z for the sample 25K DWT bulk carrier

Time window [years]	mean m.cm ²	st. dev m.cm ²	Time window [years]	mean m.cm ²	st. dev. m.cm ²	Time window [years]	mean m.cm ²	st. dev. m.cm ²
1 - 0	0.000	1727	10 - 9	245.770	2805	18 - 17	245.720	5314
2 - 1	0.000	1727	11 - 10	245.764	3087	19 - 18	245.714	5649
3 - 2	0.000	1727	12- 11	245.758	3383	20 - 19	245.707	5987
4 - 3	142.683	1757	13- 12	245.752	3690	21 - 20	245.700	6327
5 - 4	142.682	1825	14- 13	245.746	4004	22 - 21	245.694	6668
6 - 5	245.794	1932	15- 14	245.739	4325	23 - 22	245.687	7010
7 - 6	245.788	2095	16- 15	245.733	4651	24 - 23	245.680	7354
8 - 7	245.782	2302	17- 16	245.727	4981	25 - 24	245.673	7698
9 - 8	245.776	2541						

Note: The standard deviations are calculated with correlation coefficient $r = 1$ (see below).

To find the PDFs of the annual decrement of Z (further marked as $f_{\Delta}(Z_{\Delta})$), convolution integration is used. Since all PDFs of Z follow Gaussian distribution, the ‘decrement’ $f_{\Delta}(Z_{\Delta})$ also obeys Gaussian distribution with the following parameters:

$$m_{\Delta Z}(Z) = m_i - m_{i+1} \quad \sigma_{\Delta Z} = \sqrt{\sigma_{Z,i+1}^2 + 2r\sigma_{Z,i+1}\sigma_{Z,i} + \sigma_{Z,i}^2} \quad (11)$$

where $f_{\Delta}(Z_{\Delta})$ = PDF of the annual ‘decrement’ of Z, $m_{\Delta Z}(Z)$ = mean value of the decrement of Z, m_{i+1} = mean value of the $i+1$ ‘annual’ PDF of Z, m_i = mean value of i th ‘annual’ PDF of Z, $\sigma_{\Delta Z}$ = standard deviation of the decrement of Z, $\sigma_{Z,i+1}^2$ = variance of the $i+1$ ‘annual’ PDF of Z, $\sigma_{Z,i}^2$ = variance of i th ‘annual’ PDF of Z, r = correlation coefficient between section moduli Z_{i+1} and Z_i . Since the phenomenon affecting the hull girder section modulus reduction is the same (i.e., corrosion wastage) for any pair of Z_{i+1} and Z_i , r was assumed as equal to unity. However, the methodology used here allows for calculation of $\sigma_{\Delta Z}$ for any r .

When the mean values and standard deviation are calculated, the PDF $f_{\Delta}(Z_{\Delta})$ can be determined by the formula:

$$f_{\Delta}(Z_{\Delta}) = \frac{1}{\sqrt{2\pi} \sqrt{\sigma_{Z,i+1}^2 + 2r\sigma_{Z,i+1}\sigma_{Z,i} + \sigma_{Z,i}^2}} e^{-\frac{1}{2} \frac{[Z - (m_i - m_{i+1})]^2}{\sigma_{Z,i+1}^2 + 2r\sigma_{Z,i+1}\sigma_{Z,i} + \sigma_{Z,i}^2}} \quad (12)$$

or

$$f_{\Delta}(Z_{\Delta}) = \frac{1}{\sigma_{\Delta Z} \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{Z - m_{\Delta Z}}{\sigma_{\Delta Z}} \right)^2 \right] \quad (13)$$

The results of the calculations for several ‘decrement’ $f_{\Delta}(Z_{\Delta})$ are shown in Table 4. As expected, one can observe the fact that the ‘decrements’ $f_{\Delta}(Z_{\Delta})$ varies with the ship’s age. The reason is the non-linear change of the standard deviations over ship’s age. Hence, one can conclude that if probabilistic methods are applied, the simple idea implemented in Eq. (7) can be applied only in case when the PDF $f_{\Delta}(Z_{\Delta})$ does not change over ship’s service life. Obviously, this is not the case as can be observed in Table 4.

6. On the use of 3-dimensional presentation of the PDF of Z

The 3-dimensional presentation of the PDF of Z provides the opportunity of calculating the probability of Z being between any $Z_{d,u}$ and $Z_{d,l}$ during the time interval (T_u, T_l) , i.e.

$$P(Z_{d,l} \leq Z_d \leq Z_{d,u} ; T_l \leq T \leq T_u) = \frac{\int_{T_l}^{T_u} \left(C_T \int_{Z_{d,l}}^{Z_{d,u}} f_T(Z_d) dZ_d \right) dT}{\int_0^{T_s} \left(C_T \int_{b_l}^{b_u} f_T(Z_d) dZ_d \right) dT} \quad (14)$$

Note: In Eq. (14), b_u is considered as the maximum possible boundary of Z_d , b_l is considered as the minimum

possible boundary of Z_d ; T_u = upper boundary of the time 'T'; T_l = lower boundary of the time 'T'; time ' T_s ' is considered as the ship's service life.

The integral in the denominator of eq. (14) within the brackets multiplied with the coefficient C_T should always be equal to unity. In physical terms, this "unity" is equal to the ship's service life T_s . Thus, the denominator's geometric interpretation is the total space below the envelope within the given ship's service life. This space can be likened to the volume of a parallelepiped with area of the base equal to unity and height equal to T_s .

The nominator's geometric interpretation is the space below the envelope within given boundaries T_u , T_l , $Z_{d,u}$, $Z_{d,l}$ (see Fig. 15). It may be called volume of a "deformed" parallelepiped. Thus, the geometric interpretation of eq. (14) is the ratio between two volumes: the volume of the "deformed" parallelepiped and the volume of the parallelepiped with base equal to unity and height equal to T . Consequently, eq.(14) can be rewritten as

$$P(Z_{d,l} \leq Z_d \leq Z_{d,u}; T_l \leq T \leq T_u) = \frac{1}{T_s} \int_{T_l}^{T_u} \left(C_T \int_{Z_{d,l}}^{Z_{d,u}} f_T(Z_d) dZ_d \right) dT \quad (15)$$

The crosshatched area in the 'deformed' parallelepiped represent the area below the PDF of Z_d for time 'T' (in this case, $T = T_u$) within the given boundaries $Z_{d,u}$ and $Z_{d,l}$.

An example for the calculations performed with Eq. (15) is given in Fig. 16. Two types of the calculations were performed:

- Several of Z_d – ranges-of-changes were selected (i.e., 0.90-0.92; 0.90-0.94; 0.90-0.96; 0.90-0.98; 0.90-1.00, see Fig. 16). The time period was taken as equal to the service life of 25 years.
- For comparison purposes, calculations were performed for the time period $\Delta T = T_l - T_u = 10 - 20$ years for the same Z_d – ranges-of-changes as given above.

The results of the calculations are shown in Fig. 16. One can observe the fact that the probability $P(Z_d > Z_{d, \min, \text{IMO}})$ decreases with decrease of the time period and the Z_d – range-of-change.

7. Discussion

When the PDF of bending stresses in ship's hull structures is to be calculated, the still water and wave induced loads, and geometric properties of the structure should be expressed in probabilistic format. As an example, the bending stresses due to vertical wave bending moment (VWBM) is considered, the PDF of these stresses can be derived by dividing the PDF of the VWBM by the PDF of the hull girder section modulus. In current practice of Class Societies, the PDF of the VWBM is assumed to follow a Weibull distribution (Note that this PDF of VWBM is valid for the whole design life of the ship). Based on this PDF, the design VWBM in Class Societies Rules is determined as the VWBM that can be exceeded by the probability of 10^{-8} . Further, the bending stresses due to VWBM are calculated as the quotient of the design VWBM and the design hull girder section modulus.

However, when probabilistic methods are applied, it is necessary to have the hull girder section modulus, Z , expressed in probabilistic format as well. In the early publications on this subject (see references in the section of Introduction), the so-called “annual” PDF of Z is used. Later, this type of PDF is calculated for more than one year time period (for example, at year 5, 10, 15, 20, etc.) considering the detrimental corrosion effect, i.e. they are valid only at the end of the calculated time period (e.g., at the end of time period of 0 - 5 years, 0 - 10 years, 0 - 15 years, etc.).

Since the design wave-induced loads given by the Class Rules are specified for the whole ship design life, both PDFs (of the applied loads and the geometric properties of the structure) should be valid either throughout the same time-period. In this paper, the ‘time period’ PDF of hull girder section modulus is recommended as more reasonable approach in the calculation of the bending stresses since the PDF of the hull girder loads given by Class Rules is also a “time period” PDF.

8. Conclusions

The paper deals with the two ways of presenting the PDFs of hull girder section modulus: one with the so-called “annual” PDFs and another one with the “time period” PDFs. The “annual” PDFs refer only to the end of any given time period (e.g., at the end of time period of 0 - 5 years, 0 - 10 years, 0 - 15 years, etc.). The “time period” approach provides the PDFs for the whole given time period under consideration. Since the design wave-induced loads given by the Class Rules are specified for the whole ship design life, the ‘time period’ PDF of hull girder section modulus is recommended for the probabilistic analysis of wave-induced hull girder bending stresses.

The impact of with use of the two types of PDFs of hull girder section modulus on the PDFs of the wave-induced hull girder bending stresses was performed. It was found that the difference between the results obtained with the two types of PDFs of Z becomes noticeable after ship’s age of 15 years as shown in Table 2 and Fig. 14. In addition, the results also showed that the probability of exceeding (POE) of given permissible hull girder bending stress calculated using the ‘time period’ PDFs approach is always smaller than that calculated based on the ‘annual’ PDF.

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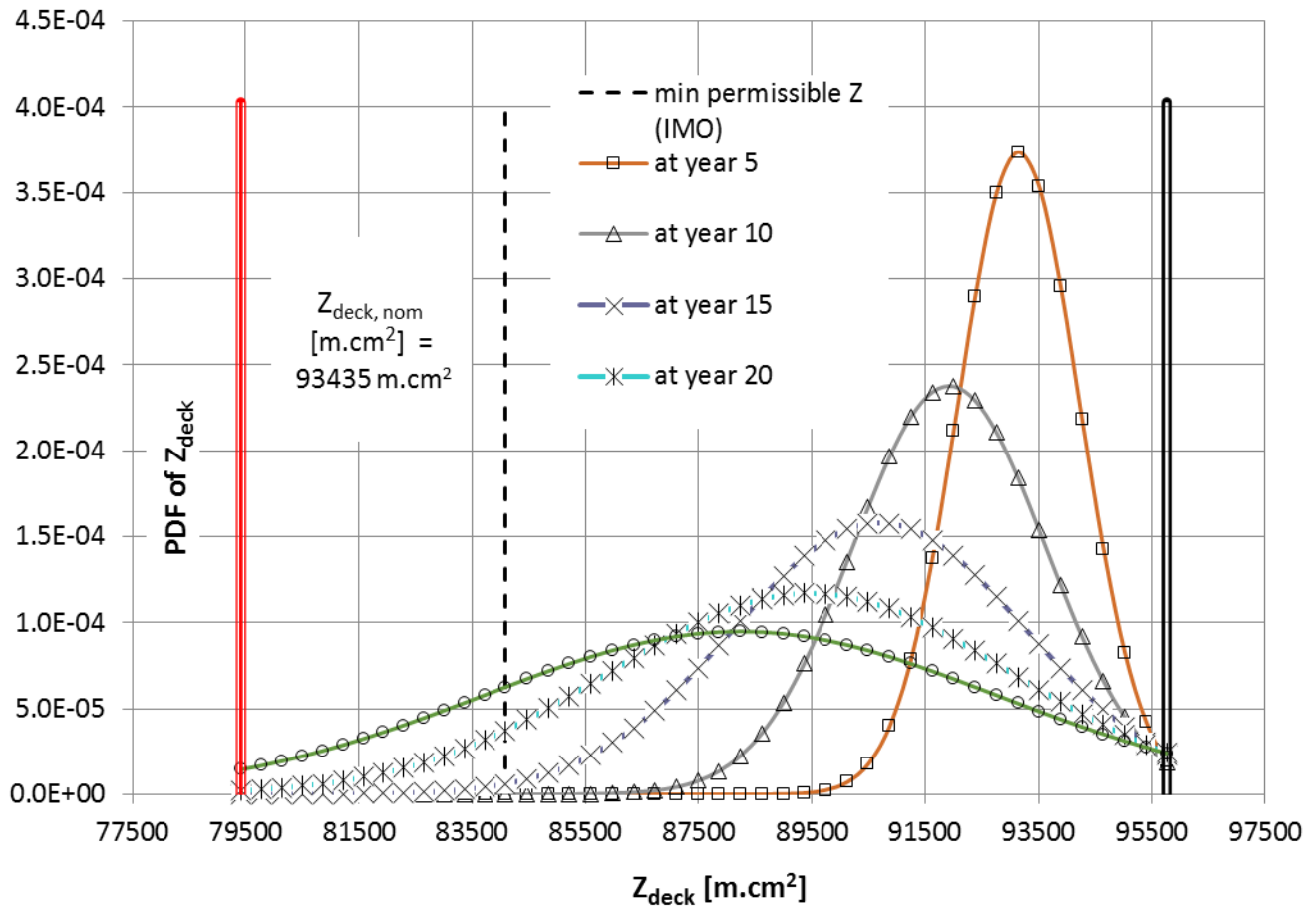


Fig. 1 Truncated Gaussian PDFs of Z of the sample 25K DWT bulk carrier at several years

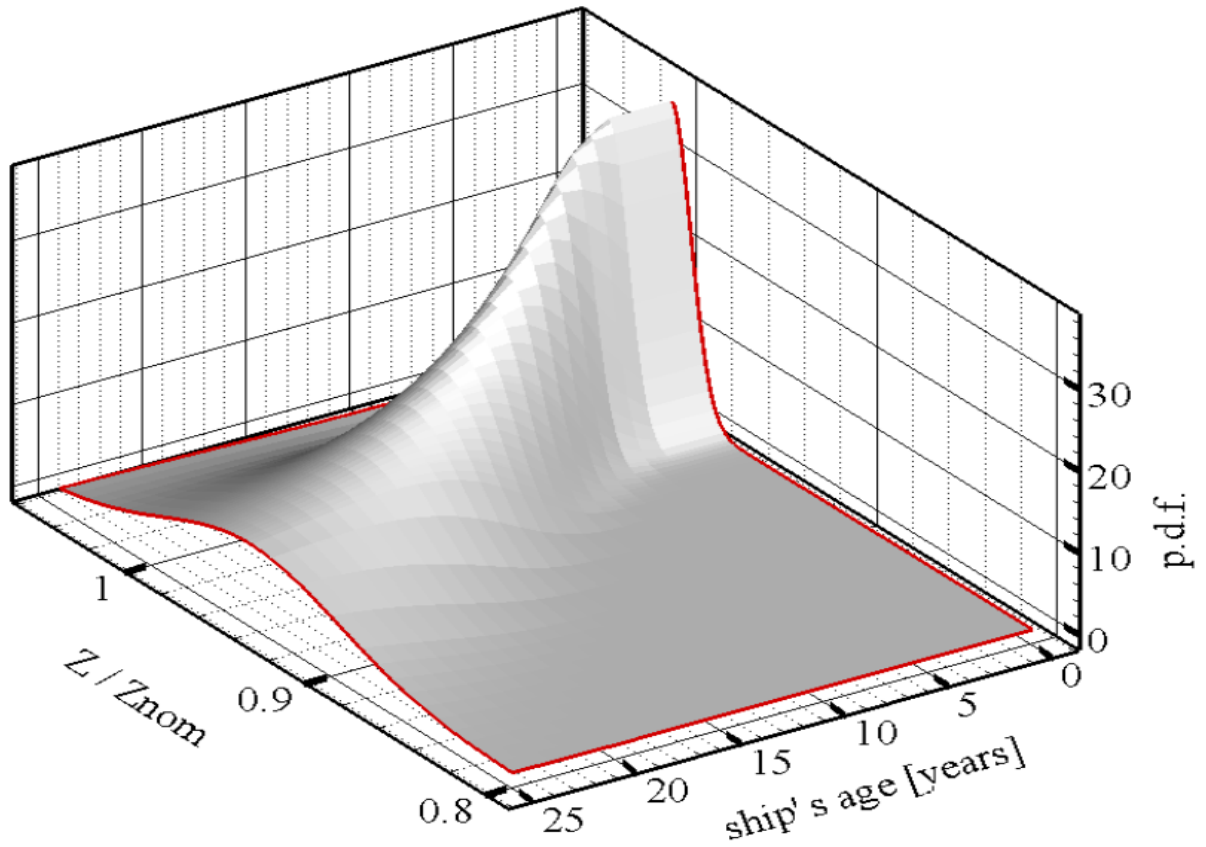


Fig. 2 Envelope of the 'annual' PDFs of Z for the sample 25K DWT bulk carrier

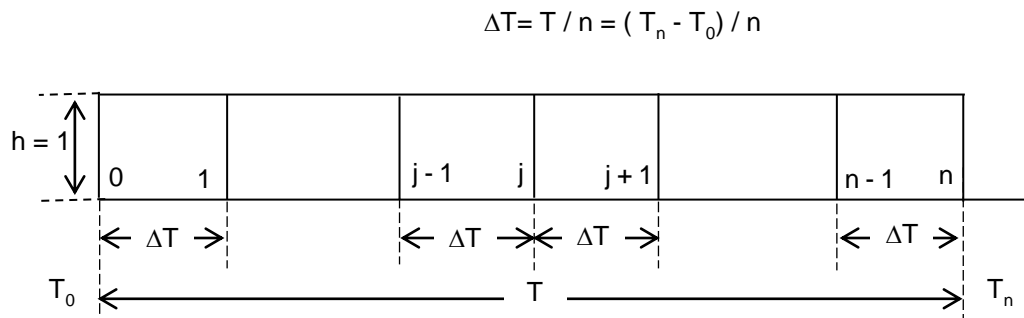


Fig. 3 Calculation of the total space below the envelope in Fig. 2 of the PDFs of Z

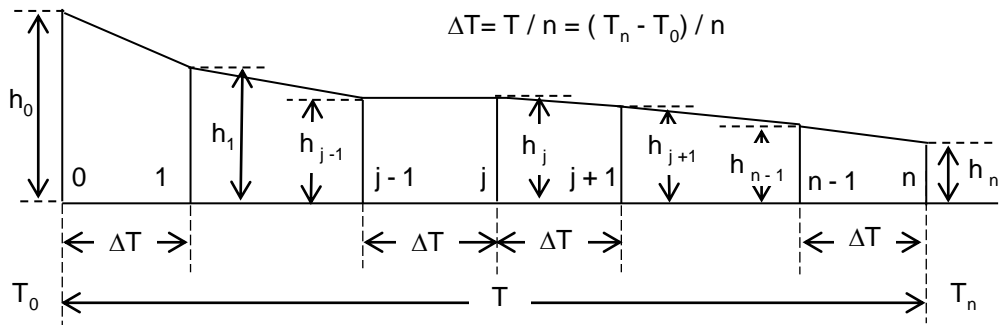


Fig. 4 Calculation of the volume below the envelope of the PDFs for any given Z

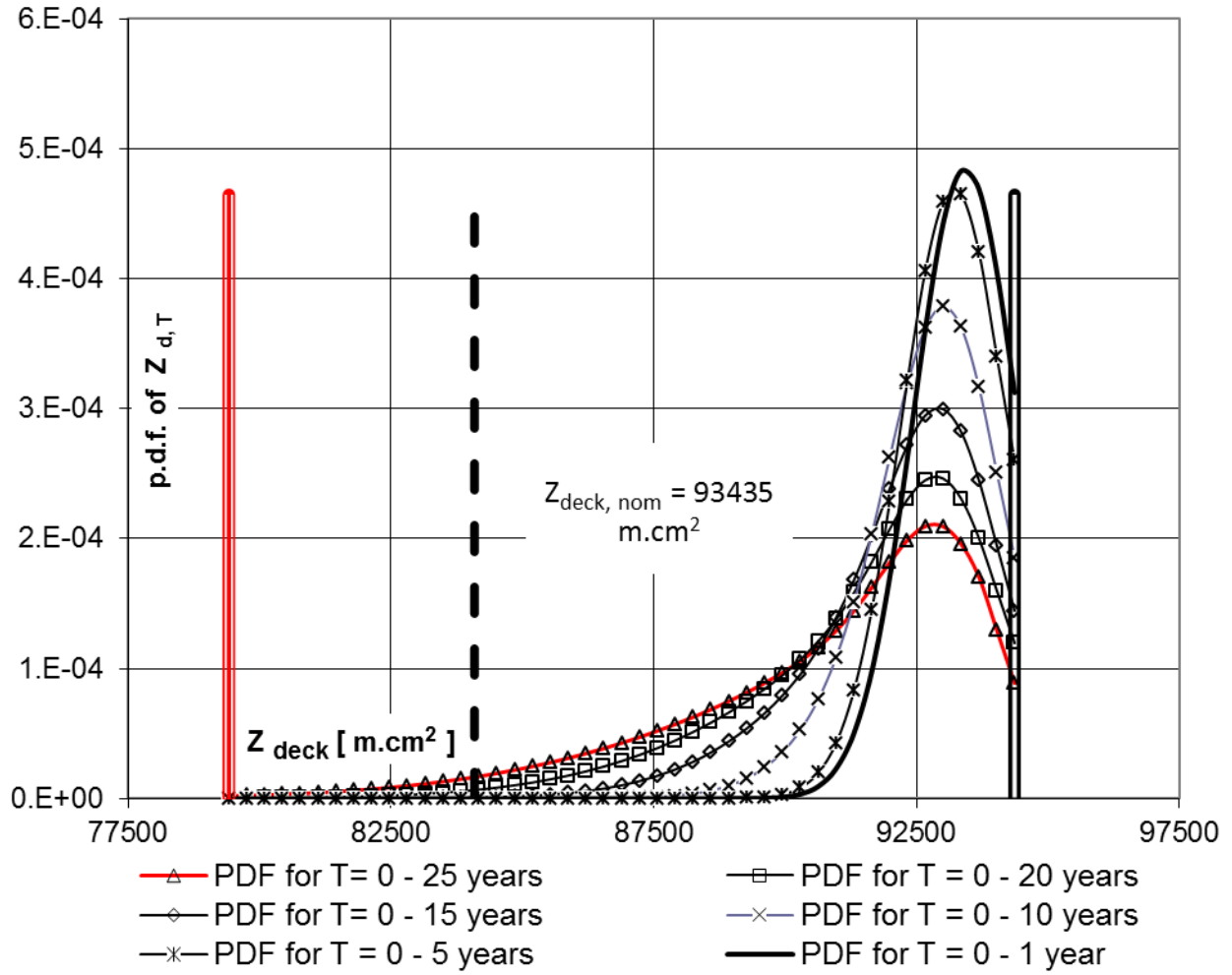


Fig. 5 Truncated PDFs of Z of the sample 25K DWT bulk carrier for given time periods

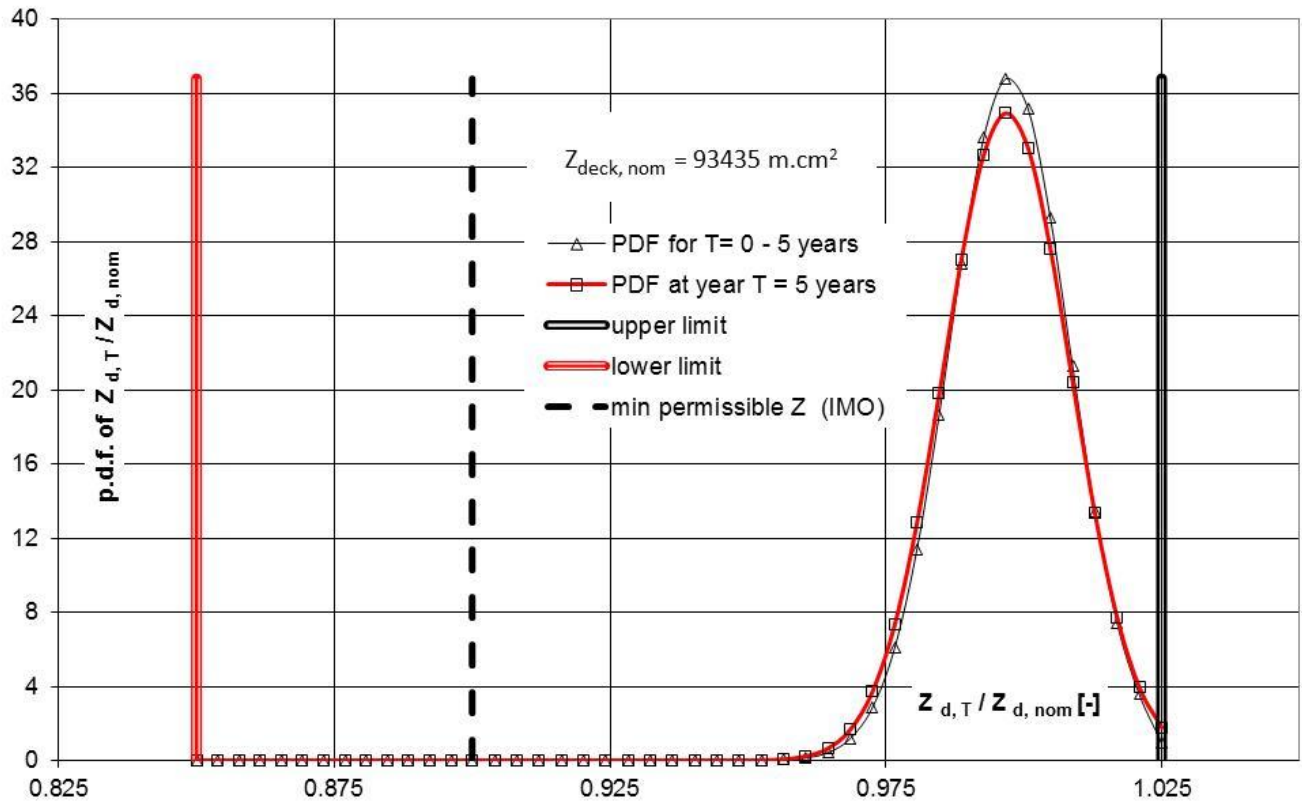


Fig. 6 Comparison between the ‘annual’ PDF at time $T = 5$ and ‘time period’ PDFs for time period 0 – 5 years

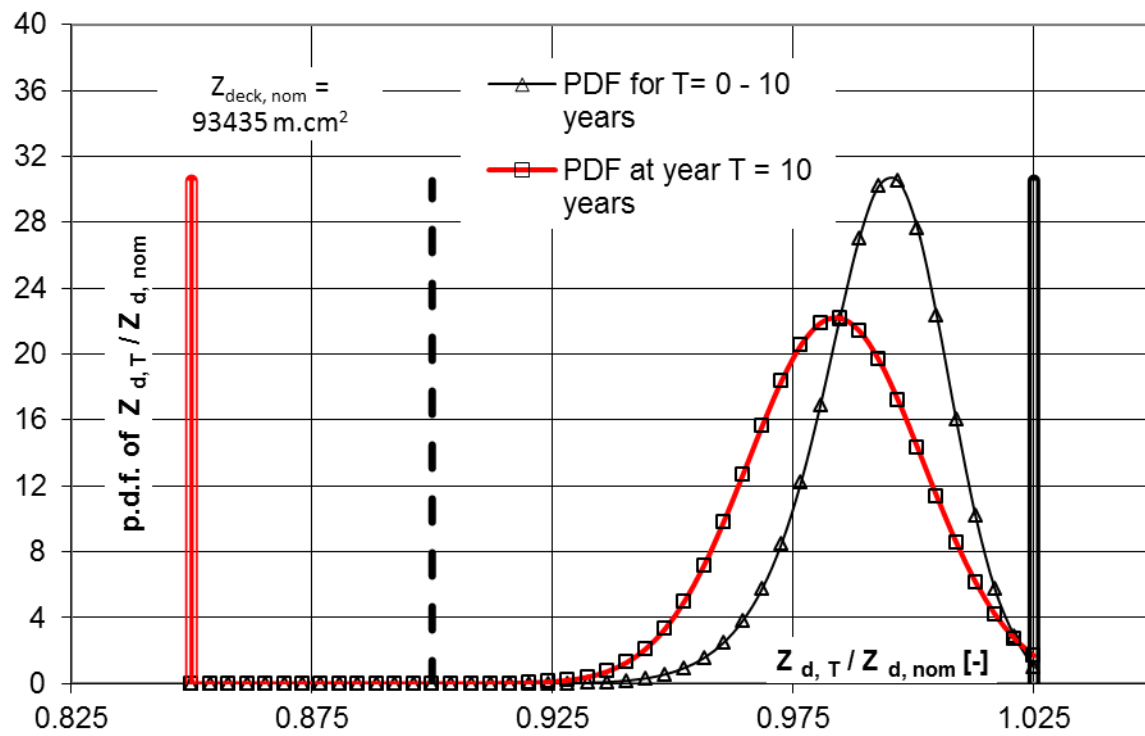


Fig. 7 Comparison between the ‘annual’ PDF at time T = 10 and ‘time period’ PDFs for time period 0 – 10 years

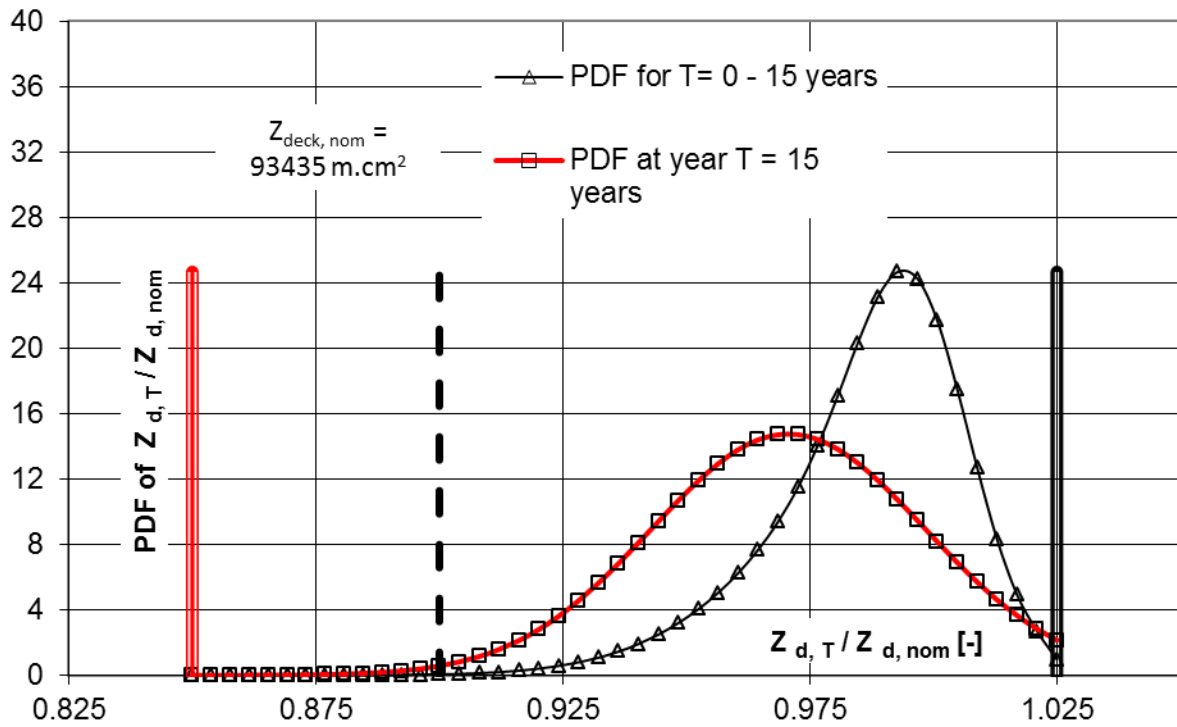


Fig. 8 Comparison between the ‘annual’ PDF at time $T = 15$ and ‘time period’ PDFs for time period $0 - 15$ years

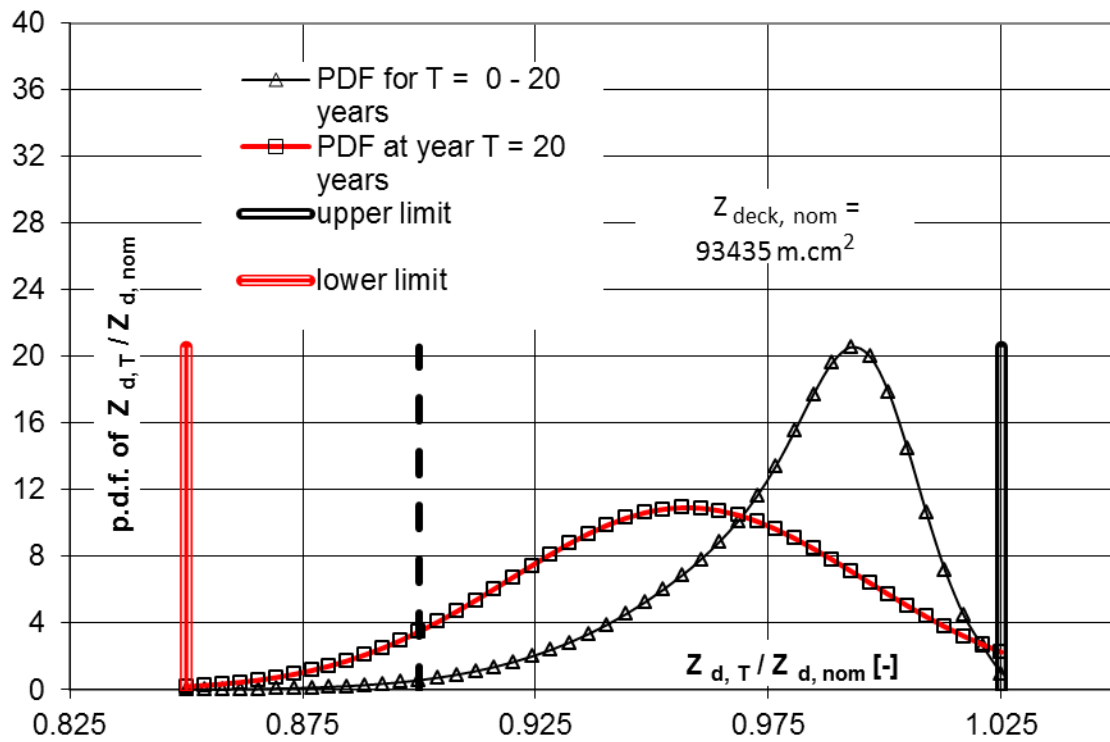


Fig. 9 Comparison between the ‘annual’ PDF at time $T = 20$ and ‘time period’ PDFs for time period $0 - 20$ years

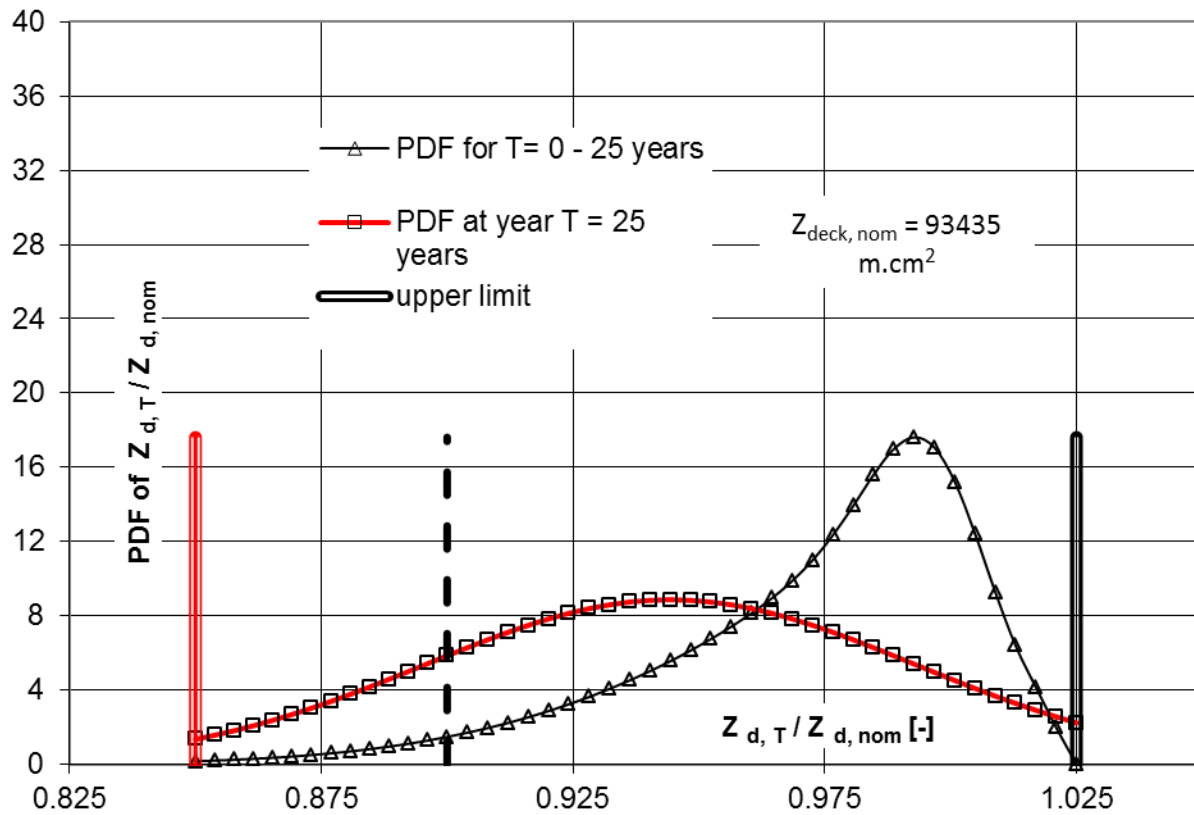


Fig. 10 Comparison between the ‘annual’ PDF at time $T = 25$ and ‘time period’ PDFs for time period 0 – 25 years

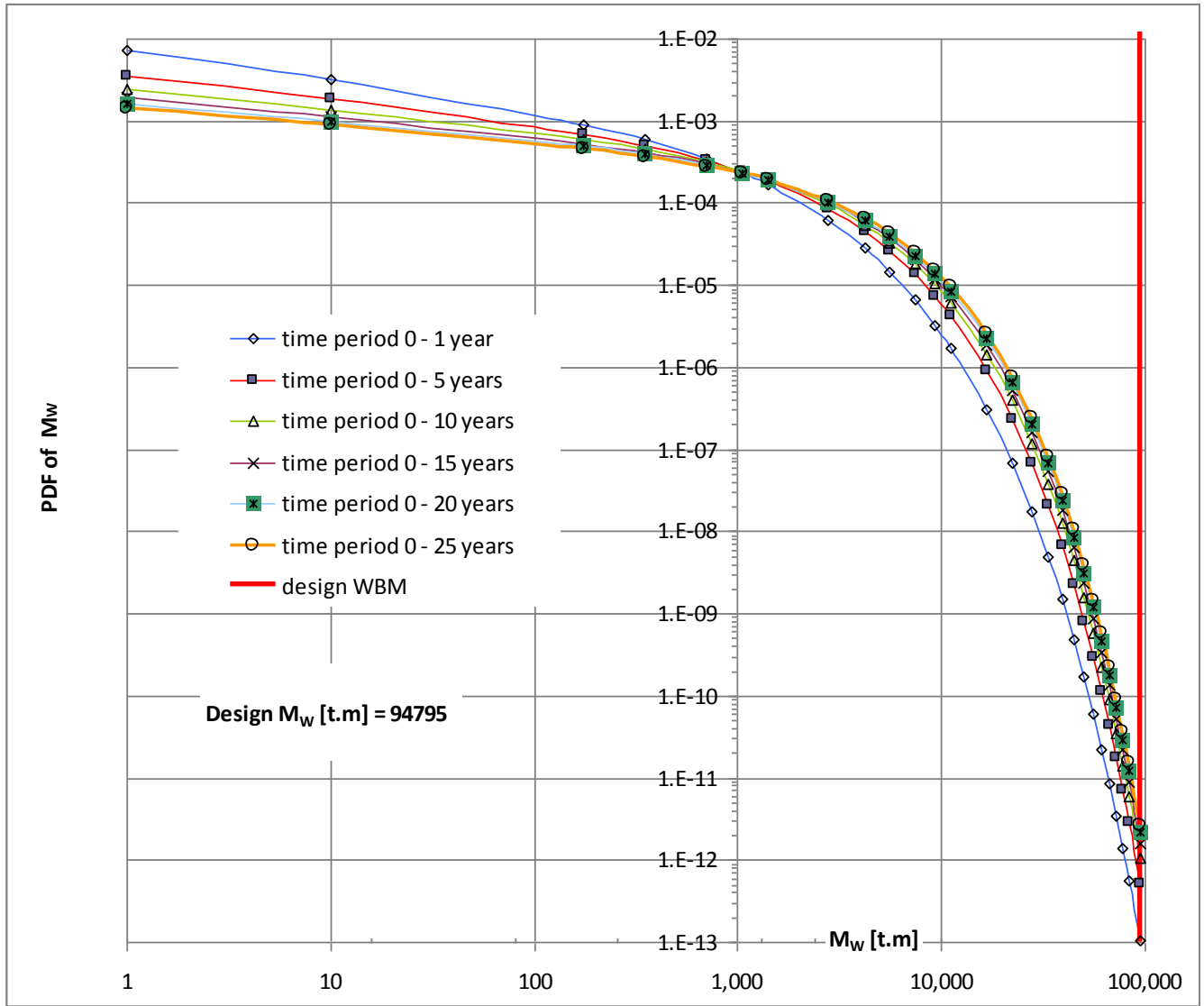


Fig. 11 PDF of VWBM for the sample 25K DWT bulk carrier for several time periods

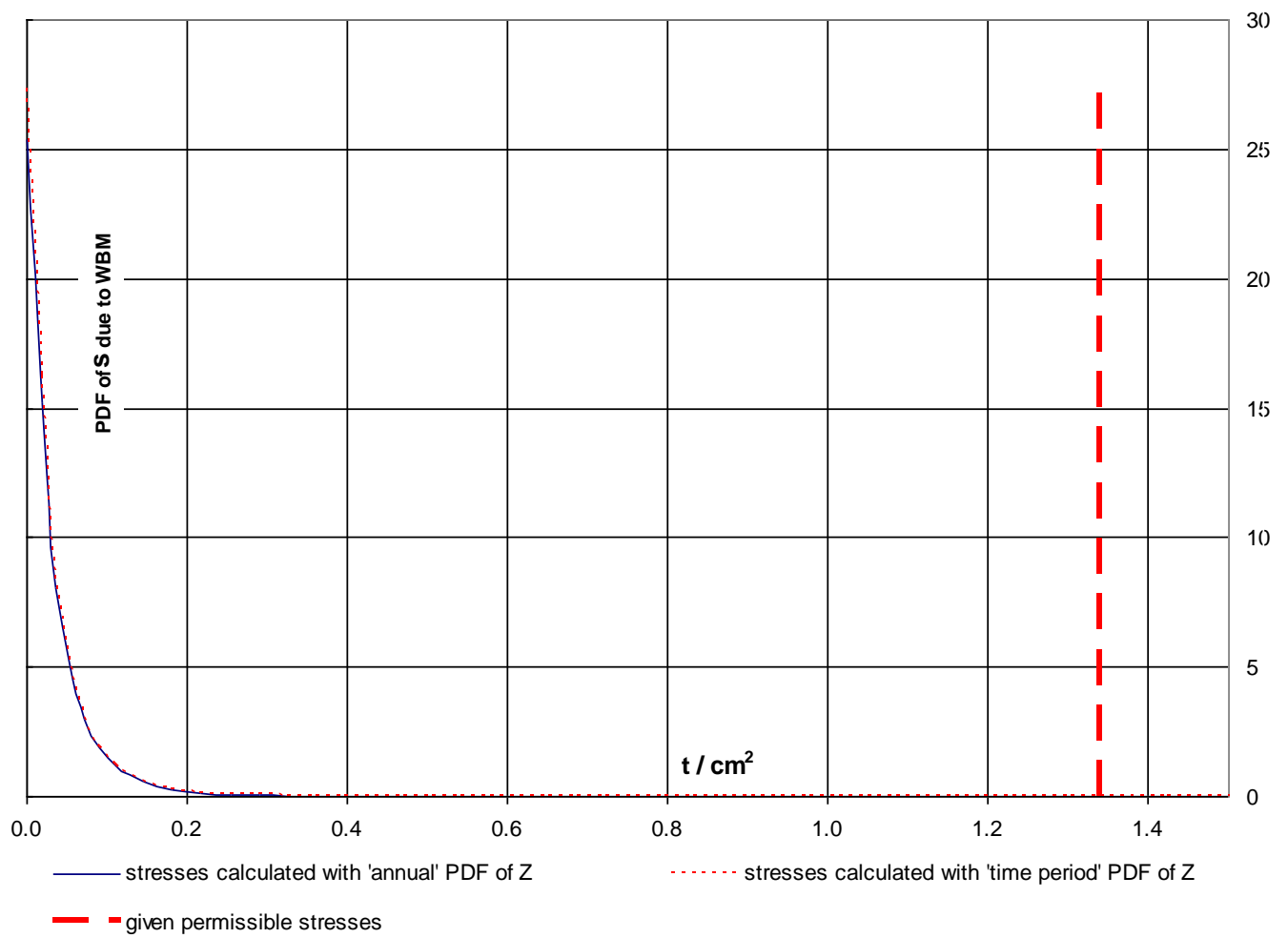


Fig. 12 PDF of bending stresses due to VWBM derived with 'annual' and 'time period' PDFs of Z

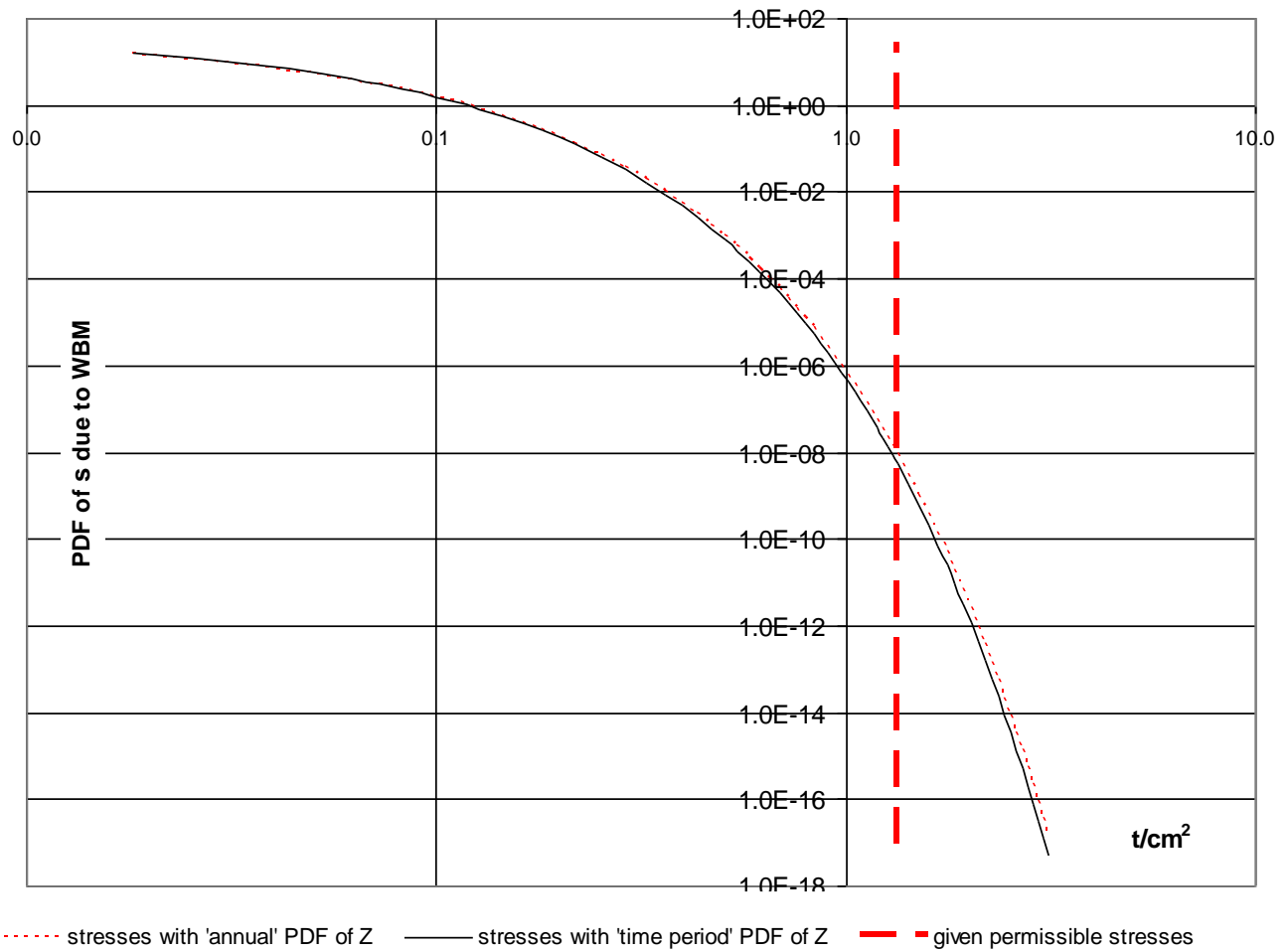


Fig. 13 PDF of bending stress due to VWBM derived with 'annual' and 'time period' PDFs of Z presented in log-scale

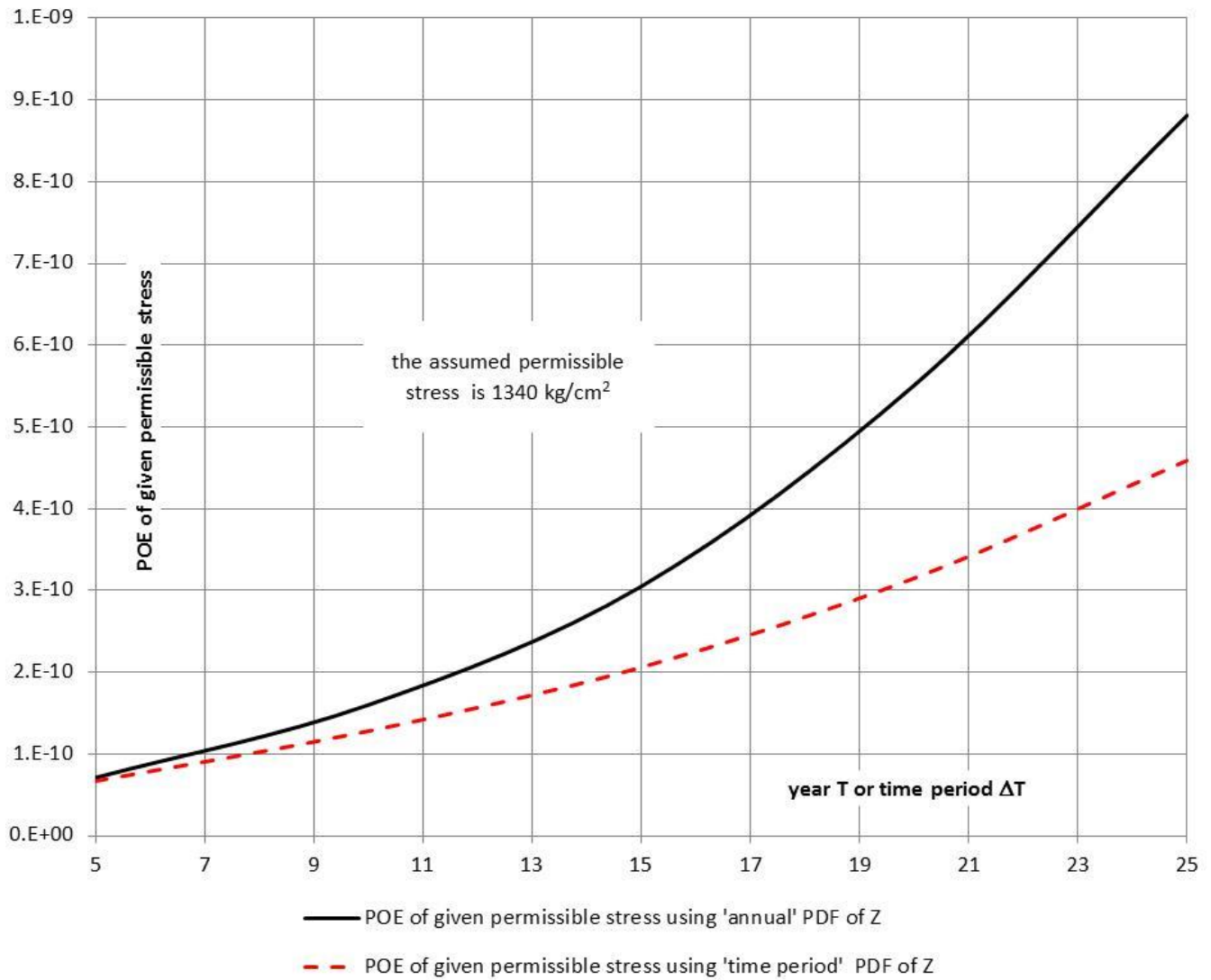


Fig. 14 POE of given permissible bending stresses due to VWBM when 'annual' and 'time period' PDFs of Z are used

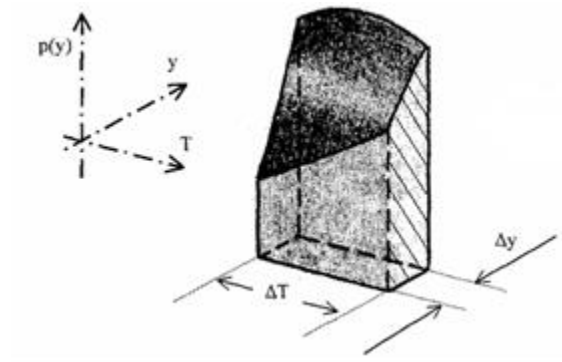


Fig. 15 Geometric interpretation of the probability $P(Z_{d,l} \leq Z_d \leq Z_{d,u}; T)$ for $T = T_u$ and probability

$$P(Z_{d,l} \leq Z_d \leq Z_{d,u}; T_l \leq T \leq T_u)$$

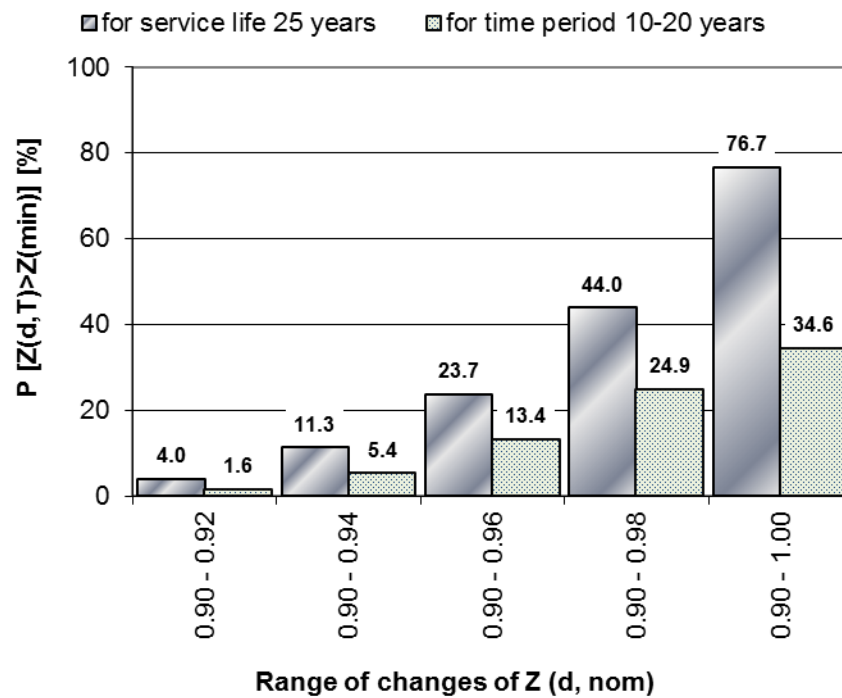


Fig. 16 $P(Z_{d,T} > Z_{\min, IMO})$ for several $Z_{d, nom}$ – ranges-of-change during the whole service life of 25 years and time period $\Delta T = 10-20$ years for the sample 25K DWT bulk carrier